Theoretical round. Problems to solve

General note. Maybe not all problems have correct questions. Some questions (maybe the main question of the problem, maybe one of the subquestions) may make no real sense. In this case you have to write in your answer (in English or Russian): «**impossible situation – ситуация невозможна**». Of course, this answer has to be explained numerically or logically.

Data from the tables (Planetary data, stars, constants, etc.) may be used for solving every problem.

The answers «**Да-Yes**» or «**Нет-No**» have to be written in English or Russian.

1. **RadioAstron.** The RadioAstron project is an international collaborative mission lead by Astro-Space Center of Russian Academy of Sciences. On July 18, 2011 a satellite, «Spektr-R», carrying a 10-m (in diameter) space radio-telescope was launched into an elliptical orbit around the Earth. Together with Earth-based radio-telescopes, «Spektr-R» works as interferometer. RadioAstron operates at the standard radio astronomical wavelengths of 1.19–1.63 cm (K-band), 6.2 cm (C-band), 18 cm (L-band), and 92 cm (P-band). Now «Spektr-R» is rotating in a very elongated orbit with a period τ = 8.3 days and a height of perigee h = 600 km from the Earth surface.
   1.1. Estimate the maximum resolving power (angular resolution in arcsec) of RadioAstron. Draw a schematic picture, explaining your choice of the situation when it may occur.
   1.2. Estimate the resolving power of RadioAstron if the target is observed in the direction of the major axis of «Spektr-R» orbit, and also draw a schematic picture.

2. **Gliese 581 g.** This celestial body in the system of the star Gliese 581 is the most Earth-like planet found outside the Solar System, and the exoplanet with the greatest recognized potential for harboring albuminous based life.
   2.1. Estimate orbital period τ of Gliese 581 g. Consider the orbit to be circular.
   2.2. Assume intelligent life resides on Gliese 581 g. The civilization uses radio-waves. Is it possible to determine the size (diameter) of the planet by observations on RadioAstron (**«да-yes»** or **«нет-no»**)? Justify the answer by calculations.

3. **Observations from Gliese 581 g.**
   3.1. What is the apparent magnitude of our Sun and **3.2.** what is the approximate constellation in which our Sun will be seen when observed from the planet Gliese 581 g?
   3.3. Estimate the angular diameter of the star Gliese 581 when observed from the planet Gliese 581 g.

4. **XVIII century. Midday.** (Dubingiai is the nearest town to the accommodation place of XVIII IAO.).
There were different systems of units of measurement in the history of science. This problem is to use historical (at present obsolete) units of measurement.
4.1. Calculate the capacity of the solar energy that in the end of the XVIII century fell on the unit of area of the territory in the outskirts of Dubingiai at midday time:
in winter, in spring, in autumn, and in summer.
The answer must be given using only the «new» physical units, which were coming into operation in those days in this area: horse-powers per square verst.

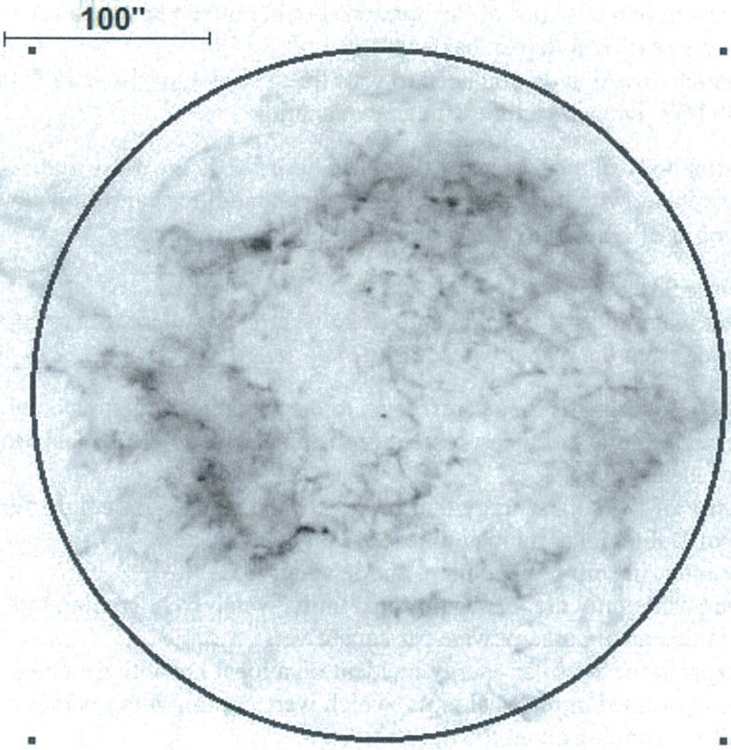
4.2. Estimate also the capacity of the solar energy incident on a local horse those times. The answer must also be expressed in physical units, which were coming into operation in those days. What can be surprising about the right answer?

5. **XXI century. Midday.** As is known, the Republic of Lithuania (see map) uses zone with winter time UT+02 and summer time UT+03. Calculate and draw a conclusion about the following:
5.1. Are there any places in Lithuania, where today (September 8, 2013) the Sun will be exactly in the south at a time when the watches of residents will show just 12:00? (**«да-yes»** or **«нет-no»**).
5.2. And in general, on the other days of the year, are there such places? (**«да-yes»** or **«нет-no»**). If "yes", then calculate in what dates, if "no", then justify it by calculations.

6. **Supernova remnant.** An X-ray image of supernova remnant (SNR) Cas A located at a distance of d = 3400 pc was obtained using Chandra Space Observatory. The negative of this image is shown in Fig. SNR. The boundaries of the SNR region are marked by a circle. The scale of the image is shown in the upper left corner of the figure. A dot located close to the center of the circle is the neutron star – the remaining core of the collapsed star. The rectangular marks outside the circle are given for the reference when determining the center of the circle.
Assume that the amount of energy released in the supernova explosion was about $E_{SN} \approx 10^{46}$ J, 1% of which drives the expansion of the remnant. The average density of the matter in the SNR is $\rho \approx 10^{-21}$ kg/m^3.
6.1. Estimate the age of the SNR Cas A.
6.2. Calculate the average velocity of the motion of the neutron star from the center of the SNR.

Теоретический тур. Рисунок к задаче 6

Theoretical round. Picture for problem 6



S N R

XVIII Международная астрономическая олимпиада

XVIII International Astronomy Olympiad

Литва, Вильнюс   6 – 14. IX. 2013   Vilnius, Lithuania

Элементы орбит. Физические характеристики некоторых планет, Луны и Солнца

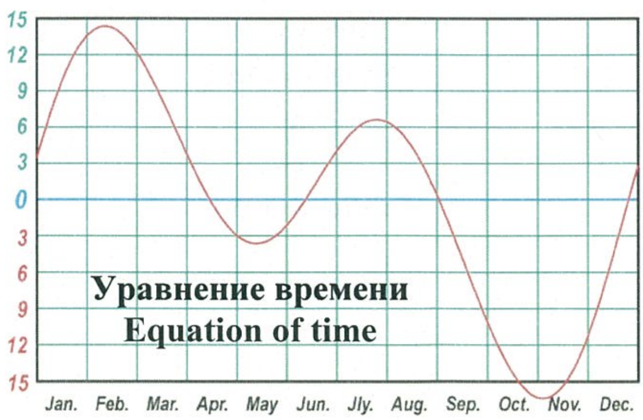
Parameters of orbits. Physical characteristics of some planets, Moon and Sun

Небесное тело, планета / Body, planet	Среднее расстояние от центрального тела / Average distance to central body		Сидерический (или аналогичный) период обращения / Sidereal period (or analogous)		Наклон орбиты, *i* / Orbital inclination, *i*	Эксцентриситет, *e* / Eccentricity *e*	Экваториальн. диаметр / Equat. diameter	Масса / Mass	Средняя плотность / Av. density	Ускор. своб. пад. у пов. / Grav. accelr. at surf.	Наклон оси / Axial tilt	Макс. блеск, вид. с Земли **) / Max. magn. From Earth **)	Альбедо / Albedo
	в *астр. ед.* / in *astr. units*	в *млн. км* / in *10^6 km*	в тропич. годах / in tropical years	в средних сутках / in days	*i*	*e*	*км* / *km*	*10^{24} кг* / *10^{24} kg*	*г/см³* / *g/cm³*	*м/с²* / *m/s²*			
Солнце Sun	$1{,}6\cdot10^9$	$2{,}5\cdot10^{11}$	$2{,}2\cdot10^8$	$8\cdot10^{10}$			1392000	1989000	1,409			$-26{,}74^m$	
Меркурий Mercury	0,387	57,9	0,241	87,969	7,00°	0,206	4 879	0,3302	5,43	3,70	0,01°		0,06
Венера Venus	0,723	108,2	0,615	224,7007	3,40	0,007	12 104	4,8690	5,24	8,87	177,36		0,78
Земля Earth	1,000	149,6	1,000	365,2564	0,00	0,017	12 756	5,9742	5,515	9,81	23,44		0,36
Луна Moon	0,00257	0,38440	0,0748	27,3217	5,15	0,055	3 475	0,0735	3,34	1,62	6,7	$-12{,}7^m$	0,07
Марс Mars	1,524	227,9	1,880	686,98	1,85	0,093	6 794	0,6419	3,94	3,71	25,19	$-2{,}0^m$	0,15
Юпитер Jupiter	5,204	778,6	11,862	4 332,59	1,30	0,048	142 984	1899,8	1,33	24,86	3,13	$-2{,}7^m$	0,66
Сатурн Saturn	9,584	1433,7	29,458	10 759,20	2,48	0,054	120 536	568,50	0,70	10,41	26,73	$0{,}7^m$	0,68

**) Для внешних планет и Луны – в среднем противостоянии.

**) For outer planets and Moon – in mean opposition.

Lietuva * Lithuania * Литва

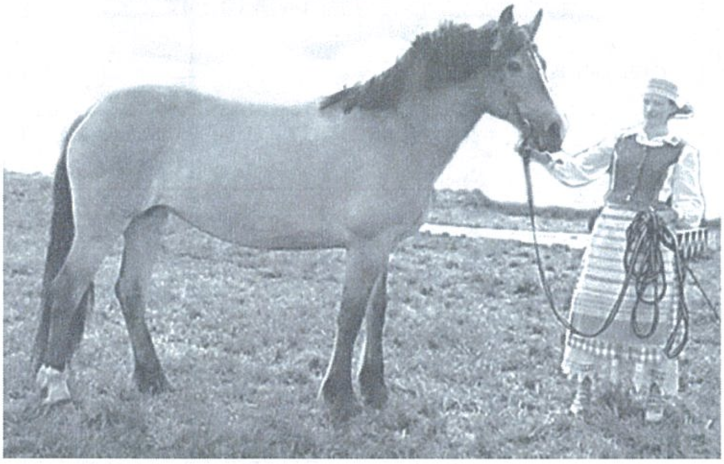


Некоторые константы и формулы
Some constants and formulae

Скорость света в вакууме, c (м/с)	299 792 458	Speed of light in vacuum, c (m/s)
Гравитационная постоянная, G ($Н·м^2/кг^2$)	$6.674·10^{-11}$	Constant of gravitation, G ($N·m^2/kg^2$)
Солнечная постоянная, A ($Вт/м^2$)	1367	Solar constant, A (W/m^2)
Параметр Хаббла, H_0 (км/с/МПк) — среднее значение / диапазон значений	71 / 50-100	mean value / diapason of values — Hubble parameter, H_0 (km/s/Mpc)
Постоянная Планка, h (Дж·с)	$6.626·10^{-34}$	Plank constant, h (J·s)
Заряд электрона, e (Кл)	$1.602·10^{-19}$	Charge of electron, e (C)
Масса электрона, m_e (кг)	$9.109·10^{-31}$	Mass of electron, m_e (kg)
Соотношение масс протона и электрона	1836.15	Proton-to-electron ratio
Постоянная Фарадея, F (Кл/моль)	96 485	Faraday constant, F (C/mol)
Магнитная постоянная, μ_0 (Гн/м)	$1.257·10^{-6}$	Magnetic constant, μ_0 (H/m)
Универсальная газовая постоянная, R (Дж/моль/K)	8.314	Universal gas constant, R (J/mol/K)
Постоянная Больцмана, k (Дж/K)	$1.381·10^{-23}$	Boltzmann constant, k (J/K)
Постоянная Стефана-Больцмана, σ ($Вт/м^2/K^4$)	$5.670·10^{-8}$	Stefan-Boltzmann constant, σ ($W/m^2/K^4$)
Константа смещения Вина, b (м·K)	0.002897	Wien's displacement constant, b (m·K)
Лабораторная длина волны Hα (Å)	6562.81	Laboratory wavelength of Hα (Å)
Длина тропического года, T (сут)	365.242199	Tropical year length, T (days)
Стандартная атмосфера (Па)	101 325	Standard atmosphere (Pa)
Ослабление видимого света земной атмосферой в зените (минимально)	19%, 0.23^m	Visible light extinction by the terrestrial atmosphere in zenith (minimum)
Показатель преломления воды при 20°C, n	1.334	Refractive index of water for 20°C, n
Момент инерции шара	$I = {}^2/_5 MR^2$	Moment of inertia of a solid ball
Объём шара	$V = {}^4/_3 \pi R^3$	Volume of a ball
Площадь сферы	$S = 4\pi R^2$	Area of sphere
π	3.14159265	π
e	2.71828183	e
Золотое сечение, φ	1.61803399	Golden ratio, φ

Данные о некоторых звёздах
Data of some stars

			RA	DEC	p	m	S C	масса mass
Солнце	Sun	☉	$0^h - 24^h$	-23°26' +23°26'	8".794	vis $-26^m.74$ bol $-26^m.82$	G2	$1\,\mathcal{M}_\odot$
Проксима Центавра	Proxima Centauri	V645 Cen, α Cen C	$14^h\ 29^m\ 43^s$	-62° 40' 46"	0".769	$11^m.05$	M5.5	$0.123\,\mathcal{M}_\odot$
Альфа Центавра	Alpha Centauri	α Cen A B	$14^h\ 39^m\ 37^s$ $14^h\ 39^m\ 35^s$	-60° 50' 02" -60° 50' 14"	0".747	$-0^m.01$ $1^m.34$	G2 K1	$1.1\,\mathcal{M}_\odot$ $0.9\,\mathcal{M}_\odot$
Бета Центавра	Beta Centauri	β Cen	$14^h\ 03^m\ 49^s$	-60° 22' 23"	0".009	$0^m.61$	B1	$21\,\mathcal{M}_\odot$
Эпсилон Эридана	Epsilon Eridani	ε Eri	$03^h\ 32^m\ 56^s$	-09° 27' 30"	0".311	$3^m.74$	K2	$0.82\,\mathcal{M}_\odot$
Глизе 581	Gliese 581	HO Lib	$15^h\ 19^m\ 27^s$	-07° 43' 20"	0".16	vis $10^m.57$ bol $8^m.0$	M3V	$0.31\,\mathcal{M}_\odot$



Меры мощности
1 лошадиная сила (л.с.) = 735,49875 Вт

Units of power
1 horse-power (hp) = 735,49875 W

Местные меры длины конца XVIII века

1 аршин (арш) = 0,711187 м
1 пядь (пд) = 1/4 аршина
1 вершок (врш) = 1/4 пяди
1 сажень (сж) = 3 аршина
1 верста (врст) = 500 саженей

Local units of length in the end of XVIII century

1 arshin (arsh) = 0.711187 m
1 span (sp) = 1/4 arshin
1 vershok (vrsh) = 1/4 span
1 sajene (sj) = 3 arshin
1 verst (vrst) = 500 sajene

Диаграмма Герцшпрунга-Рассела **Hertzsprung-Russell diagram**